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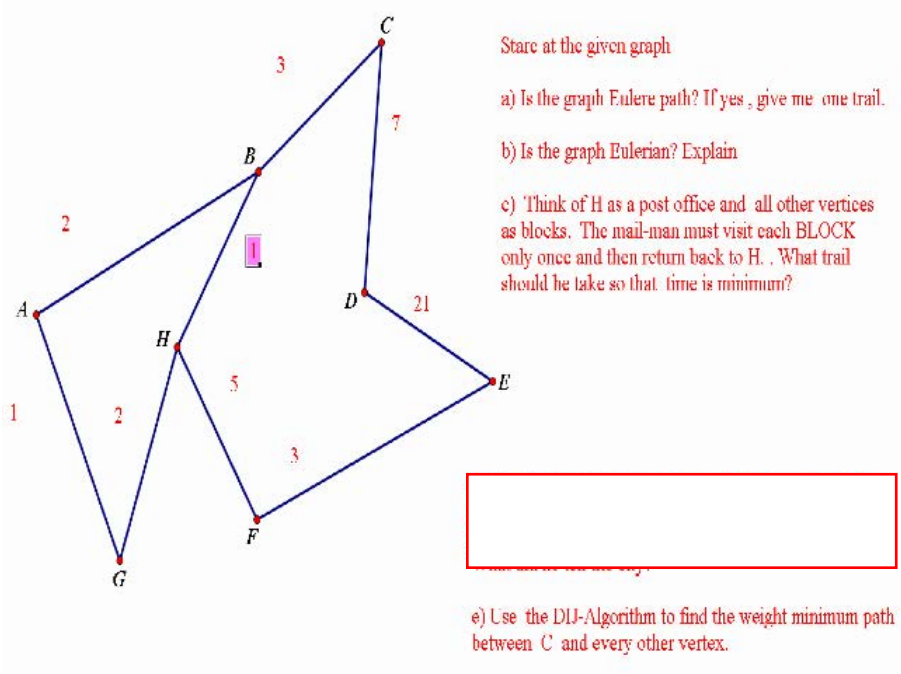
Assignment 11: MTH 213, Fall 2017

Ayman Badawi



QUESTION 1.

QUESTION 2. Draw the graph $F = \overline{K_{3,3}}$. Find $\chi'(F)$. What is $\chi'(K_{4,4})$?

QUESTION 3. Is there a tree where the vertices have degrees: 3, 3, 3, 1, 1, 1, 1, 1? if yes then draw it.

Given that 4, 2, m, n, 1, 1 are degrees of a tree? What are the values of m, n? Note $|E_T| = |V_T| - 1$ and and sum of all degrees = $2|E_T|$

Is there a graph D where $|E_D| = 6$ and $|E_{\overline{D}}| = 14$. Note that $|E_D| + |E_{\overline{D}}| = |E_{K_n}|$, where n is the order of D.

IF T is a tree, then explain to me why DIJ-Algorithm is useless :))) ? (Note if T is a tree, then there is a unique path between any two vertices!)

Let $D = K_{4,4}$. What is CL(D)? Is it clear that $CL(D)$ is Hamiltonian? So what can you say about D? Is D Hamiltonian and Eulerian?

Let $D = K_{2,3}$. What is CL(D)? Is $CL(D)$ Hamiltonian? so what can you say about D? Is D Hamiltonian and Eulerian?

Let $D = K_{2,3}$. What is CL(D)? Is $CL(D)$ Hamiltonian? so what can you say about D? Is D Hamiltonian and Eulerian?

Draw Petersen graph and find its χ'. Is Petersen graph Hamiltonian?

Draw 3-cubes and find its χ'

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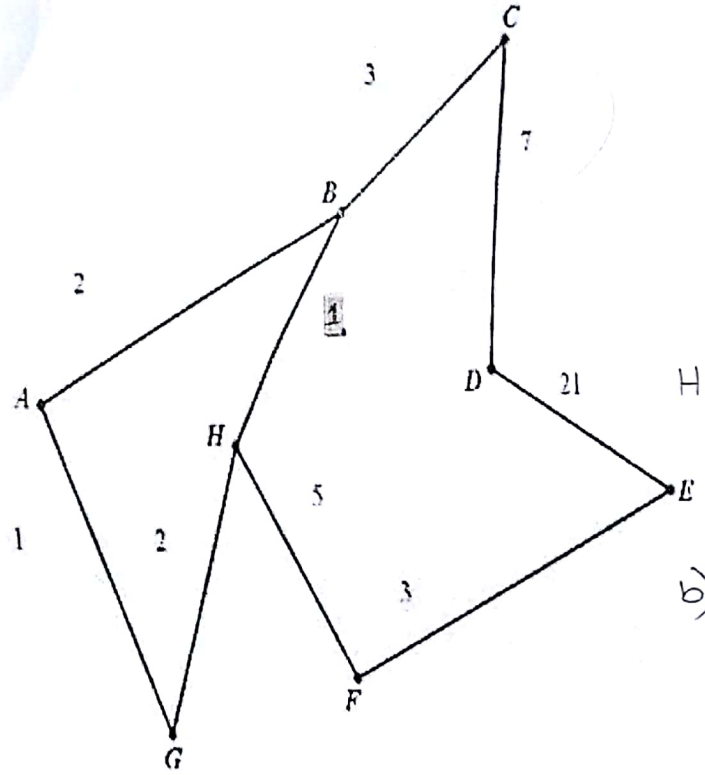
Assignment 11

Q)1)



a) Yes it is an Eulerian path because at most 2 vertices are of odd degree (B, H).

H − G − A − B − C − D − E − F − H − B

b) No, it is not Eulerian because degree degree of each vertex is not even. (deg(B) = 3, deg(H) = 3)

c)

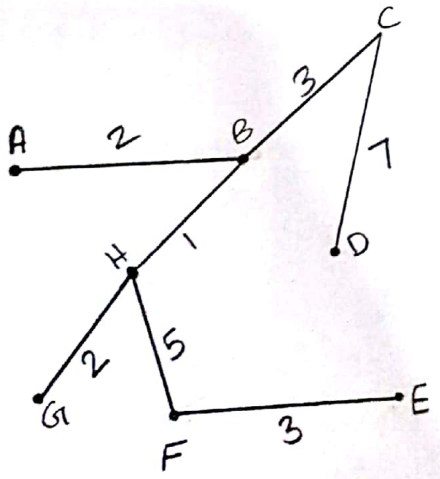
Trail 1 → H $\overset{2}{-}$ G $\overset{1}{-}$ A $\overset{2}{-}$ B $\overset{3}{-}$ C $\overset{7}{-}$ D $\overset{21}{-}$ E $\overset{3}{-}$ F $\overset{5}{-}$ H = 44

Trail 2 → H $\overset{5}{-}$ F $\overset{3}{-}$ E $\overset{21}{-}$ D $\overset{7}{-}$ C $\overset{3}{-}$ B $\overset{2}{-}$ A $\overset{1}{-}$ G $\overset{2}{-}$ H = 44

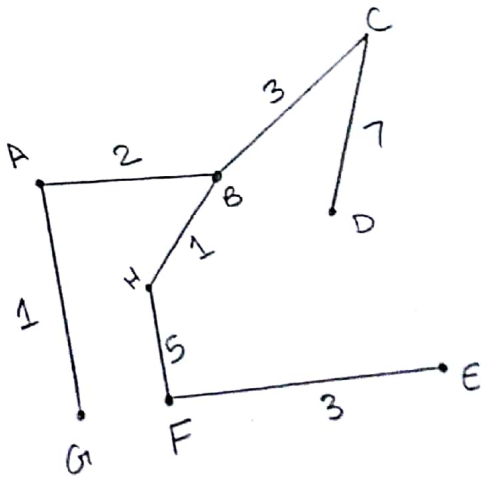
Only two possible trails. Choose either one as both have the same weight.

e)

V(s)	adj (V(s))	L(adj)	E(s)
{C}	B, D	L(B) = 3 , L(D) = 7 B enter V(s)	{C − B,
{C, B}	A, H, D	L(A) = 5 , L(H) = 4, L(D) = 7 H enter V(s)	B − H,
{C, B, H}	G, F, D, A	L(G) = 6, L(F) = 9, L(D) = 7 L(A) = 5 A enter V(s)	B − A,
{C, B, H, A}	G, F, D	L(G) = 6, L(F) = 9, L(D) = 7 G enter V(s)	H − G,
{C, B, H, A, G}	F, D	L(F) = 9 , LD = 7 D enter V(s)	C − D,
{C, B, H, A, G, D}	E, F	L(E) = 28 L(F) = 9 F enter V(s)	H − F,
{C, B, H, A, G, D, F}	E	L(E) = 12 E enter V(s)	F − E }

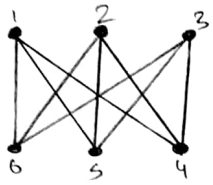


or

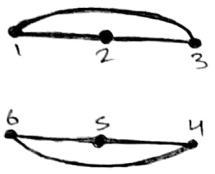


Q 2)

$= K_{3,3} \rightarrow$



$F = \overline{K_{3,3}}$

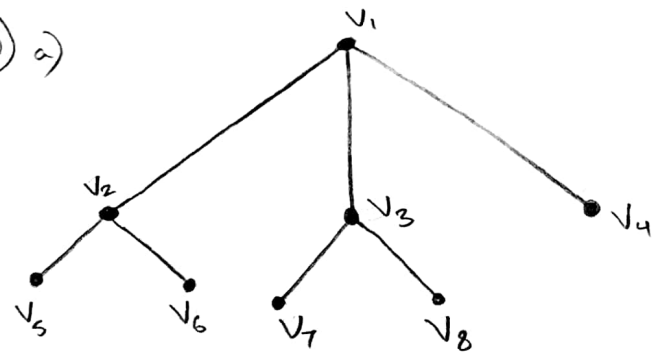


$\chi'(F) = 3$

$\chi'(K_{4,4}) = \Delta(K_{4,4})$

$= 4$

Q 3) a)

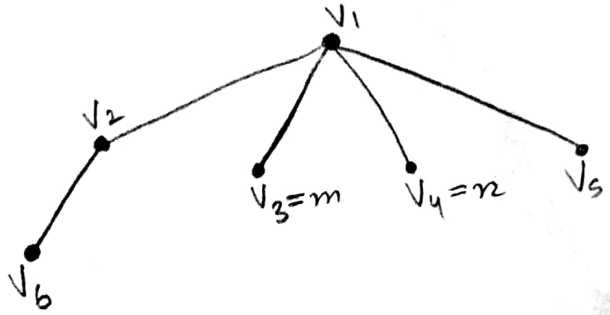


v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8

3, 3, 3, 1, 1, 1, 1, 1

b) v_1 v_2 v_3 v_4 v_5 v_6

4, 2, m, n, 1, 1



$|E_T| = |V_T| - 1$

$= 6 - 1$

$|E_T| = 5$

$\boxed{m = n = 1}$

c) $|E_D| = 6$

$|E_{\bar{D}}| = 14$

$|E_D| + |E_{\bar{D}}| = |E_{K_n}| = 20$

$$20 = \frac{n(n-1)}{2}$$

$$40 = n^2 - n$$

n is not an integer value. $\rightarrow n^2 - n - 40 = 0$

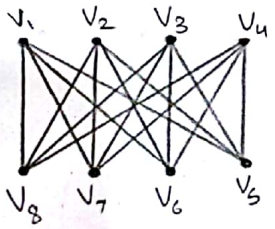
Hence there is no such graph where $|E_D| = 6$ and $|E_{\bar{D}}| = 14$

d) DIJ Agorithm is used to find a unique path of minimum weight to each vertex in a graph.

In a tree, every two vertices are connected by a unique path.

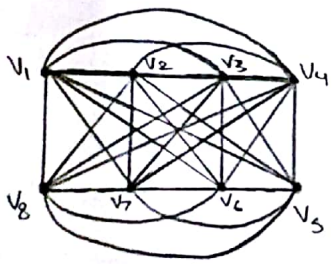
If DIJ Algorithm is used on a tree, the resulting graph will be the same original tree. Hence the algorithm is useless for trees.

e) $D = K_{4,4}$



$Cl(D)$ is Hamiltonian because $[D = K_{n,m} (n = m = 4)]$ is hamiltonian

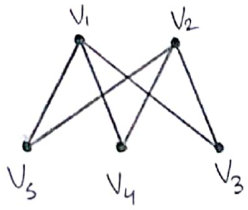
$Cl(D) =$

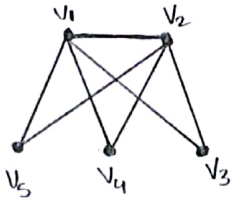


D is Eulerian because $D = K_{n,m}$ where $n, m = 4$ are even integers.

Degree of each vertex is even.

f) $D = K_{2,3}$ $\quad$ $\underline{Cl(D)}$





$\deg(u) + \deg(v) \geq n$

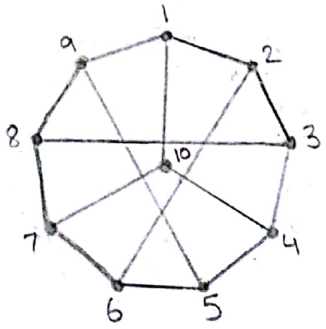
u and v are not adjacent

i) $\deg(v_1) + \deg(v_2) = 6 \geq 5$ ✓
(Add an edge)

ii) $\deg(v_3) + \deg(v_4) = 4 \geq 5$ ✗
(No edge)

- D is not Hamiltonian because Cl(D) is not Hamiltonian.
- D is not Hamiltonian because $D = K_{n,m}$ where $n \neq m$
- D is not Eulerian because $D = K_{n,m}$ where m is not an even integer

g) Petersen Graph = 3-regular graph of order 10 (connected)



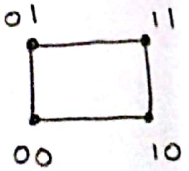
$\Delta = 3$ $\quad$ $3 \leq \chi' \leq 4$ $\quad$ $\boxed{\chi' = 4}$

Petersen Graph is not hamiltonian but it does have a hamiltonian path.

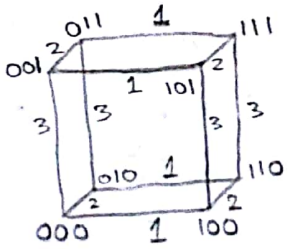
h) 3-cube

$\underline{1\text{ cube}}$

$\underline{2-\text{cube}}$



$\underline{3-\text{cube}}$



$\Delta = 3$

$\chi' = 3$